Dirty, Skewed, and Backwards: The Smectic A-C Phase Transition in Aerogel

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We study the smectic AC transition in anisotropic and uniaxial disordered environments, e.g., aerogel with an external field. We find very strange behavior of translational correlations: the *low-temperature*, *lower-symmetry* Smectic C phase is *less* translationally ordered than the *high-temperature*, *higher-symmetry* Smectic A phase, with short-ranged and algebraic translational correlations, respectively. Specifically, the A and C phase belong to the quasi-long-ranged translationally ordered " XY Bragg glass " and short-ranged translationally ordered " $m = 1$ Bragg glass " phase, respectively. The AC phase transition itself belongs to a new universality class, whose fixed points and exponents we find in a $d = 5 - \epsilon$ expansion.

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I. INTRODUCTION

Of all randomly pinned elastic media [1, 2, 3, 4] liquid crystals in aerogel [5] exhibit a phenomenon unique to themselves: anomalous elasticity. That is the scalings of their elastic energies are changed radically (specifically, by non-trivial power laws).

However, there has been no previous work on phase transitions in pinned liquid crystal systems. In this paper we remedy this by treating the smectic A to smectic C (hereafter AC) transition in an *anisotropic*, *uniaxial* disordered environment. Such an environment could be realized, e.g., by applying an electric or magnetic field to a liquid crystal in *isotropic* aerogel [6], or by stretching the aerogel uniaxially before absorbing the liquid crystal. We will hereafter refer to the special uniaxial direction as " along the applied field " or " the z-axis ".

The AC phase transition separates the two novel glassy phases discovered in reference [6]. The high temperature phase ($T > T_{AC}$) is the glassy analog of the smectic A phase of the pure problem, in that the layer normals lie, on average, along the applied field. This " random field XY smectic Bragg glass "($XYBG$) phase is in the universality class of the random field XY model. The low temperature phase is the glassy analog of the smectic C phase, in that the layers normals make an angle $\theta(T)$ with the applied field. The experimentally measurable " tilt angle " $\theta(T)$ is the magnitude of the order parameter for the transition. This phase is in the universality class of the " $m = 1$ smectic Bragg glass "($m = 1$) phase studied in [6].

We call both of these phases " glassy" because the random environment (i.e., the aerogel) destroys long-ranged translational order in both. The extent of this destruction, however, differs greatly between the two phases. Strikingly, it is the *low*-temperature, higher-symmetry, $m = 1$ phase that has *less* translational order. In the $XYBG$ phase, translational correlations are " quasi-long-ranged ", i.e., they decay as power laws with distance. In the $m = 1$ phase, these correlations are short-ranged. This leads to radically different X-ray scattering signatures in the two phases which we will now describe.

In the $XYBG$ phase, the X-ray scattering intensity $I(\vec{q})$ diverges near the smectic Bragg peaks, which occur at $\vec{q} = nq_0\hat{z}$ for all n integer, where $q_0 = \frac{2\pi}{a}$, with a the smectic layer spacing. This divergence is a power-law:

$$I(\vec{q}) \propto [(q_z - nq_0)^2 + \alpha q_\perp^2]^{\frac{-3+.55n^2}{2}}, \tag{1}$$

where α is a non-universal constant of order 1 and $q_\perp \equiv |\vec{q} - q_z\hat{z}|$. Note only the first 2 peaks ($n = 1$ and $n = 2$) actually diverge. In contrast, in the " glassy C " or " $m = 1$ BG " phase, the peaks in the X-ray scattering intensity are broad, with $I(\vec{q})$ finite for all $\vec{q}$.

As $T \to T_{AC}$ from above (i.e., on the A side), the sharp peaks disappear in an unusual way. The peaks look broad for $\vec{q}$'s sufficiently far from the Bragg peak position $nq_0\hat{z}$, while for $\vec{q}$'s sufficiently close to the peak, they diverge per eqn.(1). " Sufficiently close " means that *both* $|\vec{q}_\perp| \ll \delta q_\perp^c(n,T)$, *and* $|q_z - nq_0| \ll \delta q_z^c(T)$, where

$$\delta q_{\perp,z}^c(n,T) \propto \left(\xi_{\perp,z}^c\right)^{-\frac{n^2}{3-0.55n^2}} \tag{2}$$

with $\xi_{\perp,z}^c \sim \exp(A\,|T - T_{AC}|^{-\Omega})$, where Ω is a universal exponent calculated below and A is a non-universal constant. These predictions are illustrated in figures 1 and 2. The divergence of $\xi_{\perp,z}^c$ implies that, as $T \to T_{AC}^+$, the algebraic " spikes " on top of the broad short-ranged peaks get narrower and less intense, vanishing completely at T_{AC}. Lowering temperature further leads only to the broad peaks of the Smectic C phase. This entire scenario of sharp peaks at high temperature and broad peaks at low temperature is very counterintuitive, and unlike almost every other translationally ordered system [11].

Why is the lower-symmetry C phase *less* translationally ordered than higher-symmetry A phase? In fact, it is precisely the new broken symmetry of the smectic C phase-that is, the tilt of the layer normal-that causes this. This is because, while the energetically preferred layer normal $\hat{N}$ in the smectic A phase is *unique*-it must point *along* $\hat{z}$-there are *infinitely many* energetically preferred orientations of $\hat{N}$ in the C phase: $\hat{N}$ can lie anywhere on a cone making an angle $\theta(t)$ with $\hat{z}$. As a result, the elasticity of the Smectic C phase is *softer* than that of the A phase because rotating the layers in such a way

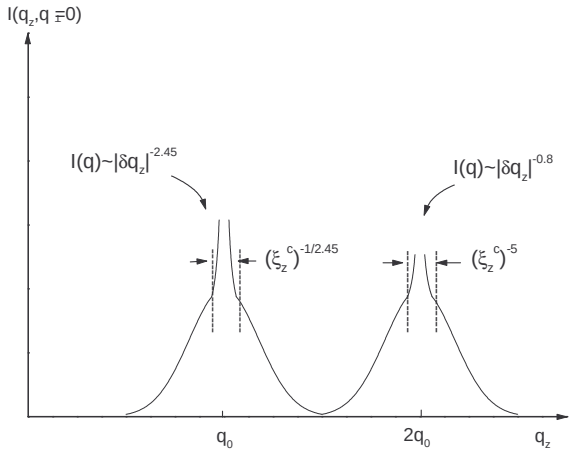

FIG. 1: The q_z-dependence of the X-ray scattering intensity for $q_\perp = 0$ in the smectic A phase. In the C phase, the sharp, power law peaks disappear, leaving only the broad scattering.

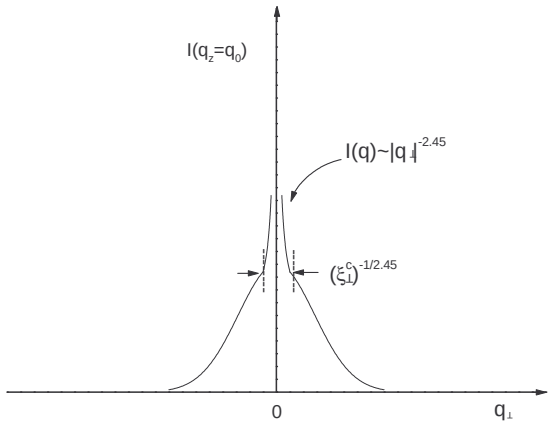

FIG. 2: The $q_\perp$-dependence of the X-ray scattering intensity for $q_z = q_0$ in the smectic A phase. Again, the sharp peak vanishes in the C phase.

that $\hat{N}$ rotates *around* this cone costs no energy. This *exact* symmetry of the elastic energy of the smectic C phase means that the direction perpendicular to the $\hat{z}$-$< \hat{N} >$ plane (where $< \hat{N} >$ is the mean of $\hat{N}$; i.e., the direction of spontaneous tilt) becomes " soft ", that is, an easy direction for layer displacements to vary in. Precisely such softness occurs (for different reasons) in the " $m = 1$ smectic " studied in [6] and, indeed, the elastic Hamiltonian for the C phase we find is identical to that studied for "$m = 1$ smectic" in [6]. Thus, we can simply transcribe the results of [6] to this problem.

In particular, positional fluctuations u of the layers about their optimal (tilted) positions obey $\overline{< |u(\vec{q})|^2 >} = TC(\vec{q})^{-1} + (\Delta_s(\vec{q})q_s^2 + \Delta_h(\vec{q})q_h^2 + \Delta_z q_z^2)C(\vec{q})^{-2}$, with $C(\vec{q}) \equiv Bq_z^2 + \gamma(\vec{q})q_h^2 + K(\vec{q})q_s^4$, where $<>$ denotes a thermal average, and the overbar denotes an average over disorder. The anomalous quantities $\Delta_{s,h}(\vec{q})$, $K(\vec{q})$ and $\gamma(\vec{q})$ obey

$$K, \gamma, \Delta_{s,h} \sim \begin{cases} q_s^{-\tilde{\eta}_K, \eta_\gamma, -\eta_{s,h}}, & q_s^{\zeta_h} \gg q_h, q_s^{\zeta_z} \gg q_z \\ q_h^{-\tilde{\eta}_K, \eta_\gamma, -\eta_{s,h}/\zeta_h}, & q_h \gg q_s^{\zeta_h}, q_h \gg q_z^{\frac{\zeta_h}{\zeta_z}} \\ q_z^{-\tilde{\eta}_K, \eta_\gamma, -\eta_{s,h}/\zeta_z}, & q_z \gg q_s^{\zeta_z}, q_z \gg q_h^{\frac{\zeta_z}{\zeta_h}} \end{cases} \quad (3)$$

where $q_h = Eq_x - Fq_z$ with $E \propto |T - T_{AC}|^{\nu_\perp\left(1-\frac{\eta_K}{2}\right)}$ and $F \propto |T - T_{AC}|^{\frac{\nu_\perp}{2}(2+\eta_t-\eta_K-\eta_c)}$, q_s is the component of $\vec{q}$ perpendicular to the $\hat{z}$-$\hat{N}$ plane, q_x is the component of $\vec{q}$ within the $\hat{z}$-$\hat{N}$ plane orthogonal to $\hat{z}$ and the critical exponents $\nu_\perp$ and $\eta_{t,K,c}$ will be given later. The *universal* exponents in (3) are given by $\zeta_h = 2 - (\frac{\eta_\gamma + \tilde{\eta}_K}{2})$, $\zeta_z = 2 - \frac{\tilde{\eta}_K}{2}$ with the η's obeying the *exact* scaling relations $1 + \eta_s = \frac{\eta_\gamma}{2} + 2\tilde{\eta}_K$ and $\eta_h = 2 + \eta_s - \eta_\gamma - \tilde{\eta}_K$. Numerical values for the η's have been obtained [6] by comparing ϵ-expansions based on two different analytical continuations of this model to higher dimensions, giving $\tilde{\eta}_K = 0.50 \pm 0.03$, $\eta_\gamma = 0.26 \pm 0.12$, $\eta_s = 0.132 \pm 0.002$ and $\eta_h = 1.372 \pm 0.12$. These predictions can be tested by light scattering, which measures $\overline{< N_i(\vec{q})N_j(-\vec{q}) >} = q_i^\perp q_j^\perp \overline{< |u(\vec{q})|^2 >}$. The form given for $\overline{< |u(\vec{q})|^2 >}$ also implies [6] short-ranged translational order and, hence, broad Bragg peaks, as described earlier.

We have studied the AC transition in an $\epsilon = 5 - d$ expansion, where d is the dimension of space, and find that there *is* a stable fixed point, implying a second-order phase transition with *universal* critical behavior. In particular, the tilt angle $\theta(T)$ obeys $\theta(T) = A\left(T_{AC} - T\right)^\beta$, where, to leading order in ϵ, $\beta = \frac{1}{2} - \frac{\epsilon}{10} + O\left(\epsilon^2\right)$.

The specific heat exponent $\alpha = -\frac{\epsilon}{10} + O\left(\epsilon^2\right)$ and the susceptibility exponent $\gamma = 0$.

The order parameter $\vec{N}_\perp$ for this transition is the projection of the smectic layer normals $\hat{N}$ perpendicular to the applied field. Above T_{AC}, real space correlations of $\vec{N}_\perp$ decay rapidly with distance, with correlation lengths ξ_z and $\xi_\perp$ parallel and perpendicular to the field respectively. Both diverge as power laws in $(T - T_{AC})$: $\xi_{\perp,z} \propto |T - T_{AC}|^{-\nu_{\perp,z}}$. We find $\nu_\perp = \frac{1}{2} + \frac{3\epsilon}{20} + O\left(\epsilon^2\right)$, $\nu_z = 1 + \frac{3\epsilon}{10} + O\left(\epsilon^2\right)$.

We also find that the system exhibits anomalous elasticity right at T_{AC} as well. Specifically, we find that, right at T_{AC}, the smectic layer bend modulus K *vanishes* as $\vec{q} \to 0$ according to the scaling laws

$$K\left(\vec{q}\right) = q_\perp^{-\eta_K} f_K\left(\frac{q_z}{q_\perp^\zeta}\right) \sim \begin{cases} q_\perp^{-\eta_K}, & q_z \ll q_\perp^\zeta \\ q_z^{-\frac{\eta_K}{\zeta}}, & q_z \gg q_\perp^\zeta \end{cases} \quad (4)$$

where the anisotropy exponent $\zeta = 2 - \frac{\eta_K}{2}$ and $\eta_K = C_K\epsilon^2 + O(\epsilon^3)$ with $C_K = \frac{32\ln\left(\frac{4}{3}\right) - 10}{225} \cong -.00353$. Note that $C_K < 0$, which implies that K *vanishes* as $\vec{q} \to 0$. The anisotropy exponent also obeys $\zeta = \frac{\nu_z}{\nu_\perp}$.

The disordering effect of the random aerogel matrix can be quantified by disorder variances Δ_t and Δ_c describing tilt and compressive stresses respectively. These variances also become anomalous, obeying $(i = t, c)$

$$\Delta_i(\vec{q}) = q_\perp^{-\eta_i} f_\Delta\left(\frac{q_z}{q_\perp^\zeta}\right) \sim \begin{cases} q_\perp^{-\eta_i}, & q_z \ll q_\perp^\zeta \\ q_z^{-\frac{\eta_i}{\zeta}}, & q_z \gg q_\perp^\zeta \end{cases} . \quad (5)$$

Unlike the similar problem of a smectic A in *isotropic* aerogel and no field, the smectic layer compression modulus B remains finite as $\vec{q} \to 0$ at T_{AC}.

The exponents η_t and η_c are given by $\eta_t = C_\Delta \epsilon^2 + O(\epsilon^3)$ with $C_\Delta = \frac{12\ln\left(\frac{4}{3}\right)-\frac{1}{3}}{225} \approx .01386$ and $\eta_c = 2 - \frac{\epsilon}{5} + O(\epsilon^2)$.

For T bigger than T_{AC} the disorder variance $\Delta_{t,c}(\vec{q}, T)$ and layer bend modulus $K(\vec{q}, T)$ are given by their $T = T_{AC}$ forms equations (4) and (5) if *either* $q_\perp \xi_\perp \gg 1$ or $q_z \xi_z \gg 1$. Otherwise (i.e., if *both* $q_\perp \xi_\perp \ll 1$ and $q_z \xi_z \ll 1$), $\Delta_{t,c} \propto \xi_\perp^{\eta_{t,c}} \propto (T - T_{AC})^{-\nu_\perp \eta_{t,c}}$ and $K \propto \xi_\perp^{\eta_K} \propto (T - T_c)^{-\nu_\perp \eta_K}$.

The critical exponents obey *exact* scaling relations:

$$\alpha = 2 - \nu_\perp\left(d - 1 + \frac{\eta_K}{2} - \eta_t\right), \quad (6)$$

$$\beta = \nu_\perp(2d - 6 + 3\eta_K - 2\eta_t)/4, \quad (7)$$

$$\Omega = \nu_\perp\left(2 - \frac{3}{2}\eta_K + \eta_t\right), \quad (8)$$

$\nu_z = \zeta \nu_\perp$ and $\zeta = 2 - \frac{\eta_K}{2}$. Note that α does *not* obey hyperscaling, due to the strongly relevant disorder.

All of these exponents can be measured experimentally. The specific heat can, of course, be measured by the usual thermodynamic measurements. The spontaneous tilt angle θ_0 can not be deduced from the position of the smectic C Bragg peak, since that peak is broad.

Fortunately, an alternative measure of θ_0 can be deduced from the dielectric or diamagnetic susceptibility tensors χ_{ij} and ϵ_{ij}. In the A phase, one of the principal axes of both tensors is along the applied field. In the C phase, this axis rotates away from the applied field due to the tipping of the layers. This rotation angle is proportional to θ_0.

The order parameter correlation lengths can be measured by light scattering, which probes fluctuations in both the dielectric(ϵ_{ij}) and diamagnetic(χ_{ij}) susceptibility tensors. The full form of the light scattering is extremely rich and complicated; we will defer a complete description of it to a future publication. Here we will restrict ourselves to pointing out that for $q_z = 0$, the light scattering intensity scales like $q_\perp^{2\eta_K - \eta_t - 4}$ for $q_\perp \gg \xi_\perp^{-1}$, is independent of $q_\perp$ for $\left(\xi_\perp^{RF}\right)^{-1} \ll q_\perp \ll \xi_\perp^{-1}$, and scales like $\frac{1}{q_\perp}$ for $q_\perp \ll \left(\xi_\perp^{RF}\right)^{-1}$, where $\xi_\perp^{RF} \propto \xi_\perp^{3 - \frac{3}{2}\eta_K + \eta_t}$ as $T \to T_{AC}^+$. Thus light scattering data should easily allow determination of $\xi_\perp(T)$ and the combination of exponents $2\eta_K - \eta_t$. Fitting the T-dependence of $\xi_\perp$ to $(T - T_{AC})^{-\nu_\perp}$ then determines $\nu_\perp$.

We now briefly sketch the derivation of our results. Our starting point is an elastic energy for the layer displacement field u, which is the *only* soft variable in the problem, since the applied field locks the nematic director $\hat{n}$ [5]. For the smectic AC transition in a *pure* (i.e., disorder-free) system, with an applied field freezing the director out, Grinstein and Pelcovits [7] showed that the appropriate elastic energy is:

$$H_{pure} = \int d^d r \left[\frac{K}{2}(\nabla_\perp^2 u)^2 + \frac{B}{2}(\partial_z u)^2 - \frac{g}{2}(\partial_z u)|\vec{\nabla}_\perp u|^2 + \frac{w}{8}\left|\vec{\nabla}_\perp u\right|^4 + \frac{D_0(T)}{2}|\vec{\nabla}_\perp u|^2\right]. \quad (9)$$

This model is very similar to that for a smectic A in the *absence* of an external field. However, because the rotational symmetry is broken due to the external field, a new term $|\nabla_\perp u|^2$, which hardens the directions orthogonal to $\hat{z}$, is generated. Since $|\vec{\nabla}_\perp u|$ is proportional to the tilt angle of the smectic layers, the coefficient $D_0(T)$ is positive in the A phase (favoring alignment of the layer normal with the applied field), and negative in the C phase (favoring tilt of the layers). Hence, by continuity, at $T = T_{AC}$, $D_0(T)$ vanishes. In what follows, we will assume that $D_0(T) \propto T - T_{AC}$ near T_{AC}.

The other terms in (9) are simply those of the elastic theory of a smectic A in *zero* field, with one crucial exception: in a smectic in *zero* field, rotation in variance requires that $g = w = B$, while for the AC in a *non-zero* field problem, at all temperatures, even at $T = T_{AC}$, where $D \to 0$ and softness is recovered, g and w are still free, because rotation invariance is still broken.

To include the effects of the quenched disorder of the aerogel, we add to the pure Hamiltonian (9) random fields coupling to u and its gradients: giving us

$$H = H_{pure} + \int d^d r \left[\vec{h}(\vec{r}) \cdot \vec{\nabla} u + V_p(u - \phi(\vec{r}))\right] \quad (10)$$

where $\vec{h}(\vec{r})$ is a quenched random field that for simplicity we take to be Gaussian zero distributed mean, and characterized by short-ranged anisotropic correlations:

$$\overline{h_i(\vec{r})\, h_j(\vec{r}')} = \left[\Delta_t \delta_{ij}^\perp + \Delta_c \delta_{ij}^z\right] \delta^d(r - r'). \quad (11)$$

The field $\phi(\vec{r})$ is also a quenched random field with only short-ranged correlations, and is uniformly distributed between 0 and a, the smectic layer spacing. The function $V_p(u - \phi)$ is periodic with period a.

The physical interpretation of the quenched random fields $\vec{h}(\vec{r})$ and $V_p(u - \phi)$ is very simple. Note that the random field $\vec{h}$ incorporates random torques and random compressions, coming from the $\perp$ and z components of $\vec{h}$, respectively. The function $V_p(u - \phi(\vec{r}))$ represents the tendency of the aerogel to pin the smectic layers in a set of random positions $\phi(\vec{r})$, modulo the smectic layer spacing a, which is why V_p is periodic in its argument.

To compute self-averaging quantities, e.g., the disorder averaged free energy, it is convenient to employ the replica "trick" that relies on the identity $\overline{\log Z} =$

$\lim_{n\to 0}\frac{\overline{Z^n}-1}{n}$. After replicating and integrating over the disorder $\vec{h}(\vec{r})$ utilizing Eq. 11 we obtain [12]

$$\begin{aligned} H[u_\alpha] &= \frac{1}{2}\int d^dr \sum_{\alpha=1}^{n}\Big[K\left(\nabla_\perp^2 u_\alpha\right)^2 + B\left(\partial_z u_\alpha\right)^2 \\ &- g(\partial_z u_\alpha)|\vec{\nabla}_\perp u_\alpha|^2 + \frac{w}{4}\left|\vec{\nabla}_\perp u_\alpha\right|^4 \\ &+ D_0(T)\left|\vec{\nabla}_\perp u_\alpha\right|^2\Big] \\ &- \frac{\Delta_t}{2T}\int d^dr \sum_{\alpha,\beta=1}^{n}\vec{\nabla}_\perp u_\alpha\cdot\vec{\nabla}_\perp u_\beta \end{aligned} \tag{12}$$

Assuming $D_0(T)$ is very small right at the phase transition the noninteracting propagator $G_{\alpha\beta}(\vec{q}) \equiv V^{-1}\langle u_\alpha(q)u_\beta(-q)\rangle$ can be easily obtained

$$G_{\alpha\beta}(q) = TG(\vec{q})\,\delta_{\alpha\beta} + \Delta_t q_\perp^2 G(\vec{q})^2 \tag{13}$$

with $G(\vec{q}) = 1/\left(Kq_\perp^4 + Bq_z^4\right)$.

We employ the standard momentum shell renormalization group (RG) transformation. The only novelty is that we will employ an infinite hyper-cylindrical Brillouin zone: $|\vec{q}_\perp| < \Lambda$, $-\infty < q_z < \infty$, where $\Lambda \sim \frac{1}{a}$ is an ultraviolet cutoff. We separate the displacement field into high and low wave vector components $u_\alpha(\vec{r}) = u_\alpha^<(\vec{r}) + u_\alpha^>(\vec{r})$, where $u_\alpha^>(\vec{r})$ has support in the hyper-cylindrical shell $\Lambda e^{-\ell} < q_\perp < \Lambda$, $-\infty < q_z < \infty$. We then integrate out the high wave vector part $u_\alpha^>(\vec{r})$, and rescale the length and long wavelength part of the fields with $\vec{r}_\perp = \vec{r}'_\perp e^\ell$, $z = z'e^{\omega\ell}$, and $u_\alpha^<(\vec{r}) = e^{\chi\ell}u'_\alpha(\vec{r}')$ so as to restore the UV cutoff back to Λ.

Evaluating the corrections δB, δK, $\delta\Delta$, δg and δu and performing the rescalings described above, we obtain the following RG flow equations to one loop order:

$$\frac{dB(\ell)}{d\ell} = \left(d-1-\omega+2\chi-\frac{3}{16}g_3\right)B, \tag{14}$$

$$\frac{dK(\ell)}{d\ell} = \left(d-5+\omega+2\chi+\frac{1}{32}g_3\right)K, \tag{15}$$

$$\frac{d\left(\Delta/T\right)(\ell)}{d\ell} = \left(d-3+\omega+2\chi+\frac{1}{64}g_3\right)\frac{\Delta}{T}, \tag{16}$$

$$\frac{dg(\ell)}{d\ell} = \left(d-3+3\chi+\frac{3}{32}g_3-\frac{9}{32}g_4\right)g, \tag{17}$$

$$\begin{aligned} \frac{dD(\ell)}{d\ell} &= \left(d-3+\omega+2\chi+\frac{9}{16}g_3-\frac{5}{16}g_4\right)D \\ &+ \frac{5}{24}K\left(g_4-g_3\right), \end{aligned} \tag{18}$$

$$\begin{aligned} \frac{dw(\ell)}{d\ell} &= \left(d-5+\omega+4\chi-\frac{3}{32}\frac{g_3^2}{g_4}\right)w \\ &+ \left(\frac{3}{8}g_3-\frac{15}{32}g_4\right)w, \end{aligned} \tag{19}$$

where $g_2 \equiv \Delta\left(B/K^5\right)^{\frac{1}{2}}C_{d-1}\Lambda^{d-5}$, $g_3 \equiv \left(\frac{g}{B}\right)^2 g_2$, $g_4 \equiv \left(\frac{w}{B}\right)g_2$, $\epsilon = 5-d$. These RG flow equations have two fixed points: one preserving rotation invariance ($g_3 = g_4$), which is unstable; and one with $g_3^* = 0$, $g_4^* = \frac{32}{15}\epsilon$, which is stable and controls the second-order phase transition. Analyzing the RG flows around the stable fixed point in the standard way leads to the critical properties, exponents, and scaling relations described earlier.

In summary, a theory of smectic $A-C$ phase transition in a field in disordered media is developed. We found the critical exponents to first order in the $\epsilon = 5-d$ expansion. In addition, we have made experimentally testable predictions for the elasticity and fluctuations of this system in both phases, and at the transition.

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